Black Holes and Strings

Plan:

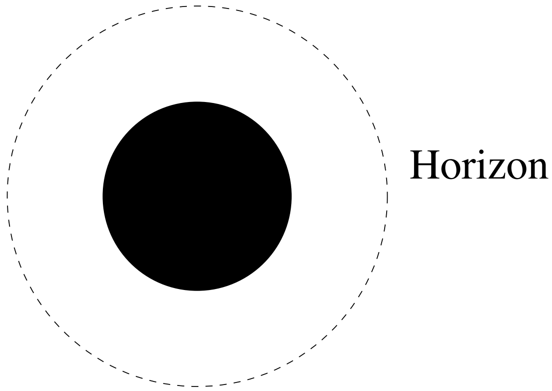
- Black holes
- String theory
- Black holes and strings

BLACK HOLES

Black holes are very heavy objects.

Their gravitational attraction is so large that even light cannot escape a black hole.

Black holes arise as classical solutions of the equations of motion of Einstein's **general theory of relativity** – the theory that makes the theory of gravity consistent with the principles of **special theory of relativity**.



For any black hole we can draw an imaginary surface around it such that no object inside the surface can ever escape to the outside world.

This surface is called the event horizon.

General theory of relativity is based on the principles of **classical mechanics**.

– All quantities are measurable.

However with the advent of **quantum mechanics** we know that this is not true.

We cannot simultaneously measure all quantities.

Question: How does the description of a black hole change when we take into account the effect of quantum uncertainties?

A complete answer to the story would require making gravity consistent with quantum theory.

This is a difficult task.

However in the 70's Bekenstein and Hawking tried a somewhat different approach.

Treat the black hole as a classical solution as before, and treat everything else using the principles of quantum mechanics.

Upon taking into account quantum effects one finds that the picture of the black hole described earlier gets modified.

In its interaction with other objects the black hole behaves as a thermal object with definite temperature, entropy etc.

In particular its entropy is given by

$$S_{BH} = C \times A$$

A: Area of the event horizon

C: A universal number that can be calculated in terms of various known constants of nature

The thermal nature of the black hole leads to some puzzles.

It has been known for a long time that for ordinary objects the thermal nature can be associated with lack of information.

For example if we take some gas confined in a box, it is practically impossible to find the exact quantum state that the gas is in.

– requires detailed knowledge of the state of every molecule.

So we average over all possible states of the gas molecules subject to some constraints based on what we can measure.

Examples of such constraints:

1. Total energy of the gas

2. Total electric charge carried by the gas.

etc.

We shall call these quantities **macroscopic** quantities.

This procedure, called **statistical mechanics**, leads to the laws of thermodynamics.

In particular entropy of a system has a very simple interpretation in statistical mechanics:

$$S_{stat} = k_B \ln \Omega$$

k_B: Boltzmann's constant

Ω (**degeneracy**) : Number of quantum states available to the system for a given set of macroscopic quantities.

Thus the entropy of a system directly measures the lack of information.

Caution: One should not confuse this lack of information with the quantum uncertainty principle.

The quantum uncertainty principle makes it impossible to measure all quantities simultaneously even in principle.

In computing entropy we count different possible values of only those quantities which can be measured according to the uncertainty principle.

The ‘statistical uncertainty’ comes because it is difficult in practice, – not in principle, – to know the exact state of a large system.

Let us now return to black holes.

Question: Does the entropy of a black hole have a similar statistical interpretation?

In the original analysis of Bekenstein and Hawking the black hole entropy did not arise from statistical mechanics, but from the analysis of the behaviour of quantum particles around a black hole solution.

This led Hawking to suggest that the thermal behaviour of the black hole is not due to an averaging procedure used for convenience, but implies an intrinsic lack of information beyond what quantum uncertainty principle gives us.

This is an apparent violation of the rules of quantum mechanics and calls for a serious study.

Such a study can be undertaken only if we can treat all systems, including the black hole, quantum mechanically.

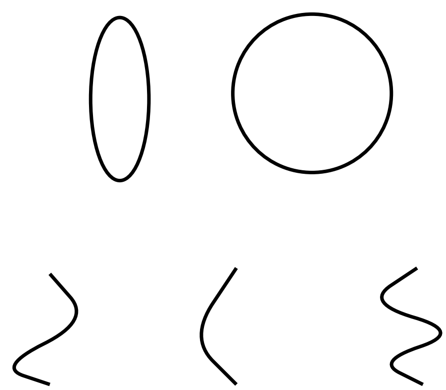
$\longrightarrow$ requires us to find a quantum theory of gravity.

This is where **string theory** comes in.

STRING THEORY

– a theory that attempts to give a unified description of all elementary particles and the forces operating between them.

Basic postulate: Different elementary particles are different vibrational states of a string.



Typical size of a string $\sim 10^{-33}$ cm.

This is much smaller than the length scale that can be probed by any present day experiment

($\sim 10^{-16}$ cm.)

Thus to the present day experimentalists the elementary string states will appear to be point-like.

We want to formulate a theory of strings consistent with the principles of

1. Quantum mechanics.

2. Special theory of relativity.

One finds that one of the vibrational states of such a string has the properties expected of a graviton, – the mediator of gravitational force.

$\longrightarrow$ String theory automatically contains gravity.

On the other hand the requirement of satisfying the laws of quantum mechanics and special theory of relativity also puts strong constraints on the theory.

Dimension of space = 9,   time=1

(instead of 3)

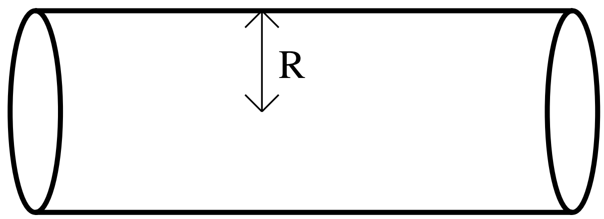
The problem of having six extra dimensions can be resolved using an old idea known as

Compactification

Take 6 of the 9 dimensions to describe a small compact space instead of infinite flat space.

When the size of the compact space is smaller that the resolution of the most powerful microscope, the space will appear to be 3 dimensional.

Example: Consider the surface of a cylinder of radius R.



If R is smaller than the resolution of the most powerful microscope, no direct experiment can distinguish this world from a one dimensional world.

The same principle can be used to make a 9 dimensional world appear 3 dimensional.

The equations of string theory puts constraints on what kind of compact space we can use in string theory.

There are many possible choices.

These different choices are like different phases of the theory, just like ice, water and steam are different phases which would arise in a theory of H_2O molecules.

In some phases, i.e. some choices of the six dimensional compact space, the effective 3 dimensional theory looks very similar to the theory that describes our world.

In particular the vibrational states of the string contains mediators of not only gravity, but of other forces which are observed in nature.

This gives us hope that string theory will eventually be able to provide us with a unified theory of the elementary constituents of matter and their interactions.

We shall not attempt to describe our progress in this direction today.

Instead I shall focus on the black hole entropy puzzle.

The reason that this issue can be addressed without knowing which phase of string theory describes our universe is that the puzzle in universal and arises in all phases.

So we can pick any suitable phase and analyze this issue.

In nature we observe only a few elementary particles.

But the full string theory contains much more than just a few elementary particles and their interactions.

A string can be viewed as a collection of infinite number of harmonic oscillators.

Each of these harmonic oscillators have infinite number of quantum states.

Thus quantization of a string gives an infinite tower of states.

Besides these states, string theory also contains other 'composite states' known as D-branes and / or solitons.

Some of these states should describe the elementary particles we observe in nature.

But most of these states have mass so large that they are not observable in present day experiments.

Thus existence of these states is not inconsistent with the fact that we only observe a finite number of elementary particles in present experiment.

These massive objects can provide important theoretical tool in understanding quantum gravity.

The degeneracy of these states grow rapidly with mass.

Thus it seems natural to define a 'statistical entropy' associated with these massive objects:

$$S_{stat}(M,\vec{Q}) = k_B \ln \Omega(M,\vec{Q})$$

$\Omega(M,\vec{Q})$ = degeneracy of states with a given mass M and charges $\vec{Q} = (Q_1, Q_2, \ldots)$.

Since string theory includes gravity, any object of very large mass produces strong gravitational field and behaves like a black hole.

One can assign an 'entropy' to these black holes via the Bekenstein-Hawking formula:

$$S_{BH}(M, \vec{Q}) = C\, A$$

A: area of the event horizon

C: A calculable constant

Question: Is $S_{BH} = S_{stat}$?

For a wide class of black holes in a wide class of phases the answer has been found to be

yes

This not only provides us with a statistical interpretation of the black hole entropy, but also provides us with an important test of internal consistency of string theory.

This formula is also remarkable due to the fact that it relates a geometrical quantity like the area of the event horizon to the result of a counting problem.

What next?

The comparison between S_{BH} and S_{stat} is usually done in the limit when the charge and mass of the black hole is large.

This simplifies calculation on both sides.

On the black hole side the calculation simplifies because the black hole size is large compared to the typical size of a string, and we can calculate the area of the event horizon ignoring 'stringy effects' and quantum effects.

On the other hand when the charges are large, we can calculate S_{stat} by using some approximate formula for the degeneracy that holds for large charges.

This is often simpler than computing the complete degeneracy.

However in order to test string theory to finer details we should try to extend these tests beyond the large charge limit.

For this we need to address two separate issues.

1. We need an algorithm for computing S_{BH} taking into account stringy / quantum effects.

2. We also need to know how to calculate the degeneracy of states to greater accuracy so that we can compute S_{stat} more accurately.

There has been much progress on both fronts.

1. We now have exact result for S_{stat} for several black holes.

2. The $S_{BH} = S_{stat}$ relation has been shown to hold beyond the leading order in many examples.

Summary

Black hole thermodynamics poses many puzzles.

Solving these puzzles is a challenge for any theory of quantum gravity.

String theory has so far successfully met this challenge, thereby establishing itself as a strong candidate for a quantum theory of gravity and everything else.